Equations (11) and (12) may be manipulated to yield

$$\alpha(P + jQ) = j\omega(U_m - U_e) \quad (17)$$

$$\beta(P + jQ) = \omega(U_T - U_z) \quad (18)$$

in which

$$U_m = U_{mT} + U_{mz} \quad (19)$$

$$U_e = U_{eT} + U_{ez} \quad (20)$$

$$U_T = U_{mT} + U_{eT} \quad (21)$$

$$U_z = U_{mz} + U_{ez}. \quad (22)$$

The quantities U_m and U_e are in general pure real for lossless, passive systems. However U_T and U_z are in general complex. Thus, the following waveguide theorems result.

a) For propagating waves ($\Gamma = j\beta$)

$$P = \frac{\omega}{\beta} \operatorname{Re}(U_T - U_z)$$

$$Q = \frac{\omega}{\beta} \operatorname{Im}(U_T - U_z)$$

$$U_m = U_e.$$

b) For evanescent waves ($\Gamma = \alpha$)

$$P = 0$$

$$Q = \frac{\omega}{\alpha}(U_m - U_e)$$

$$\operatorname{Re}(U_T) = \operatorname{Re}(U_z)$$

$$\operatorname{Im}(U_T) = \operatorname{Im}(U_z).$$

c) For complex waves ($\Gamma = \alpha + j\beta$)

$$P = 0$$

$$Q = \frac{\omega}{\alpha}(U_m - U_e)$$

$$\operatorname{Re}(U_T) = \operatorname{Re}(U_z)$$

$$\alpha \operatorname{Im}(U_T - U_z) = \beta(U_m - U_e).$$

These theorems apply, in general, to waveguides containing lossless, passive Tellegen media. For the special case of bidirectional waveguides, all of the transverse and longitudinal pseudo energies are pure real and the theorems presented above reduce to the more familiar ones [1]–[4].

In summary, we have demonstrated that the bidirectional waveguide theorems need not be derived by considering a standing wave and can be derived directly from the transverse and longitudinal components of Maxwell's equations for a single waveguide mode. Furthermore, these theorems may be generalized to include nonbidirectional waveguides containing the Tellegen medium. The results for each of the three different types of waves give relations similar to those obtained for bidirectional waveguides. As a consequence of the possibility of complex pseudo-energy terms for nonbidirectional waveguides, the interesting properties are revealed that propagating waves can carry reactive power as well as real power, while evanescent waves and complex waves carry reactive power but no real power.

Because of the general application of these theorems to nonbidirectional waveguides as well as bidirectional, we feel an appropriate name by which they should be referred to is the "waveguide power-mode theorems." The use of the combination "power-mode" stresses that the theorems deal with power relationships for a *single* waveguide mode.

The method of derivation employed here can be used to extend the work of Laxpati and Mittra [1] on periodic and open waveguides, again without employing the artifice of setting up standing waves. Also, waveguide systems that contain active media such as electron beams can also be handled in a similar manner. Active waveguides will be the subject of a future treatment by the authors.

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Plotting the Electromagnetic Field by Power Dissipation

Useful information on the electromagnetic field traveling inside a waveguide can be obtained by detecting, with temperature sensitive paints, the dissipated power on diaphragms placed transversely to the guide axis.

In the study of some ferrite structures, a technique was used which allowed the experimental verification of some properties of the electromagnetic field by detecting the RF power dissipation on certain characteristic regions [1], [2].

This correspondence deals with an extension of the preceding technique to obtain some useful and straight information about the field traveling inside a waveguide.

When a thin dissipative sheet is placed transversely to the axis of a waveguide, if only one mode is propagating, the dissipated power on the sheet follows exactly the distribution of the Poynting vector, for TE modes. By neglecting the transverse flow of power on the sheet, which is certainly permissible if the conductivity is not too low, it can give, with good approximation, the Poynting vector distribution for TM modes. If more than one mode is propagating inside the guide, while simple inspection of the power dissipation plot does not in general allow a detailed modal analysis, detection of more than one mode is certainly possible.

To reveal the dissipated power, a temperature sensitive paint has been used, which at a specific temperature (in this case 43°C) melts and becomes transparent. In this way the points where the melting of the paint starts, indicate the position of the maxima of the Poynting vector. But if the dissipative diaphragm is realized in such a way that a large

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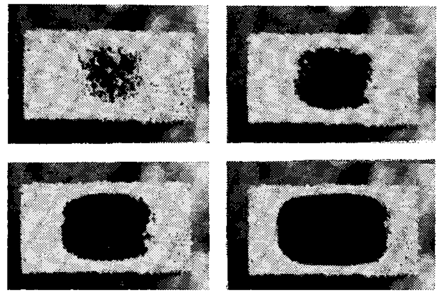

Fig. 1. Dissipation plot on a thin graphited paper placed at the open-terminal section of standard X-band waveguide, fed with a 5 watt klystron at 10 GHz. The photographs were taken at intervals of about 10 seconds.

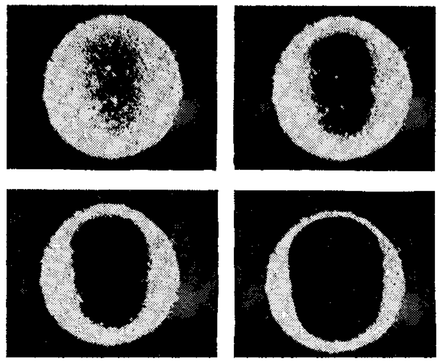

Fig. 2. Dissipation plot on a thin graphited paper placed at the open-terminal section of a circular guide—in which only the fundamental mode can propagate—fed with a 5 watt klystron at 10 GHz. The photographs were taken at intervals of about 10 seconds. The slight asymmetry is probably due to some nonhomogeneity of the graphited sheet.

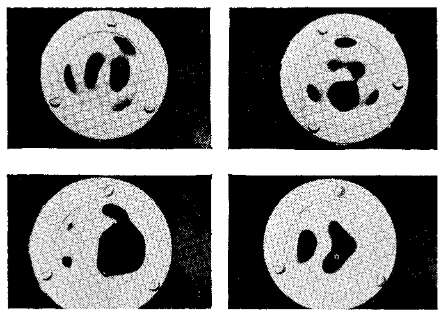

Fig. 3. Typical dissipation plots that can be observed on a thin graphited paper placed at the open-terminal section of a multimode circular guide fed with a 5 watt klystron at 10 GHz for different values of polarization of the exciting field. The inner diameter of the guide is 53 mm.

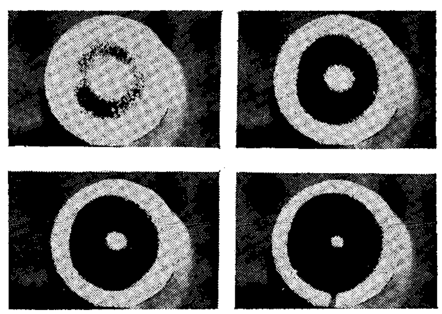

Fig. 4. Dissipation plot on a thin graphited paper placed at the open-terminal section of a circular guide fed with 5 watt klystron at 10 GHz and with the walls made with an isolated copper wire of 0.2 mm of diameter tightly wound. The inner diameter of the guide is 53 mm. The photographs were taken at intervals of about 10 seconds.

cross thermal resistance is obtained (for instance, using thin graphited paper), by following the paint melting in time, more information about the distribution of the incident power can be obtained.

The following experiments on some well-known structures have been performed. The first experiment refers to two guides of rectangular and circular cross section, respectively, in which only the fundamental mode can propagate (Figs. 1 and 2).

For the second experiment we have realized a multimode guide, joined asymmetrically to an exciting standard rectangular guide, which can be rotated around its axis, in order to obtain a variable polarization of the impressed field. Thereafter, the dissipative sheet has been placed on the terminal section and the dissipation has been visualized for many polarizations of the exciting field.

Many configurations of the kind shown in Fig. 3, have been obtained showing that more than one mode is propagating.

Finally, in order to obtain the propagation of the TE_{01} mode only, using a well-known technique [3], the metallic continuous wall has been replaced by a wall made by an isolated copper wire tightly wound.

The results of Fig. 4 were obtained by performing the experiment with such a guide.

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A Note on "Submillimeter Wave Harmonic Mixing"

In their recent correspondence, Strauch et al. [1], described two ways to prove the generation of harmonics by a crystal harmonic generator. In their experiments they used two millimeter-wave klystrons, one of them was swept with Δf, the other acted as local oscillator operated in CW. The two outputs were mixed at the diode of a crossed-waveguide device. There, a fundamental or a harmonic mixing, respectively, took place. The difference frequency signals produced in this way were amplified in a 30 MHz IF amplifier. The detected video output was displayed on an oscilloscope. They observed upper and lower sidebands. The distances between these were 60, 30, 20, 15 · · · MHz, in general 60 MHz/n. Now they stated that all these beats were produced by harmonics, moreover they claimed to have observed more than 20 harmonics from a 72.9 GHz klystron. We have strong objections to these statements. This also holds for the papers of Murai [2], [3], who claimed to have observed harmonics of the order of 14 in a similar experiment.

If the rectified current of the diode contains harmonics of the frequency nf_1-mf_2, then frequencies of $(n+m)f_1$ should also be present with the same order of magnitude. With $n=m=20$ this means that the 40th harmonic would have to be generated in the diode with about the same intensity as the 20th harmonic IF. Besides the fact that this is very unlikely, we can give a more probable explanation of such beats as a result of our experiments. The fundamental beat frequency f_1-nf_2 (where $n=1$ for normal mixing with two nearly equal frequencies, and $n=2$, 3, 4 for harmonic mixing) can generate its own harmonics in the diode. Since this frequency is low the conversion efficiency is much higher than for microwave frequencies, and on the oscilloscope it cannot be distinguished from the higher frequency harmonic beats. Instead of saying the frequency nf_1-mnf_2 is generated, we state that the nth of f_1-mf_2 is generated in the diode.

Only the interesting experiment with the OCS gas absorption cell shows clearly the existence of harmonics up to the sixth.

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Authors' Reply[1]

Schulten and Stoll are correct when they point out that harmonics of the fundamental beat frequency can produce the IF beats detected. However our experiments show the beats are observed independent of this low-frequency harmonic generation. For example, a signal of comparable magnitude is detected when a 70 GHz signal is mixed with a 59.503 GHz signal using a 60 MHz IF amplifier. In this case, as pointed out by Frenkel,[2] there are also numerous possibilities for producing the beat other than the 17th harmonic of 70 mixing with the 20th harmonic of 59.503. One of these is the 20th harmonic of the 10.497 GHz fundamental beat mixing with the 3rd harmonic of 70 GHz. Whether these possible combinations are more likely than the high frequency harmonic mixing is not readily apparent.

If the fundamental beat occurs at microwave frequencies it is difficult to prevent these currents from producing harmonics, but if the low-frequency currents described by Schulten and Stoll are prevented from entering the external circuit, these currents will be small since they must pass through the millimeter wave by-pass capacitance. By noting the effects on the output as the impedance for these low-frequency currents is controlled, we conclude that the harmonic beats are produced by microwave or millimeter wave harmonic generation and mixing.

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[1] Manuscript received July 12, 1966.
[2] Private communication.

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There may be a possibility of distinguishing between beats of the high-frequency harmonics and the harmonics of the fundamental beat frequency by, for instance, controlling the low-frequency impedance, as mentioned in the reply, but as long as the microwave harmonics are not radiated and detected separately the proof of their existence will be dubious.

[3] Manuscript received September 1, 1966.

Propagation in Cylindrical Waveguide Containing Inhomogeneous Dielectric

Considerable attention is being paid to the problem of propagation of electromagnetic waves in inhomogeneous media. Little appears to have been published in systems with cylindrical symmetry. Typical of a numerical approach is the paper by Clarricoats and Oliner[1] where an equivalent circuit is used to represent the inhomogeneous waveguide. On the other hand, Yamada and Watanabe[2] have solved analytically a special case where the dielectric constant is a parabolic function of the radius. The solution obtained was restricted to circularly symmetrical modes, and was exact for TE modes only.

This note deals with another special case where the dielectric constant varies inversely as the square of the radius. Exact analytic solutions are obtained for all possible modes, and conditions for the existence of these modes are established.

As far as we were able to ascertain this is the only analytic solution for both TE and TM type modes in inhomogeneous media in cylindrical coordinates which is known to date. As such, it may serve as a guide in looking for other cases amenable to analytic approach. However, the problem solved is a very special

Manuscript received June 14, 1966.
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